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COMMENT

Storing correlated patterns in Hopfield's model

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Abstract. We study the retrieval properties of Hopfield's model when there is a finite correlation between two of the memorised patterns.

The Hopfield model for associative memory (Hopfield 1982) has recently been thoroughly studied as a device for storing uncorrelated patterns (Amit *et al* 1987). The situation with correlations has received comparatively little attention, although it is the one we are ultimately interested in.

In this comment we obtain the $T=0$, $N\to\infty$ phase diagram of a network of N Ising spins $S_i=\pm1$ with couplings

$$J_{ij}=\frac{1}{N}\sum_{\mu=1}^{p}\xi_i^\mu\xi_j^\mu \tag{1}$$

where $\xi_i^\mu=\pm1$ are quenched independent random variables, except for ξ^1 and ξ^2, whose overlap

$$Q=\frac{1}{N}\sum_{i=1}^{N}\xi_i^1\xi_i^2 \tag{2}$$

may be finite. As usual we assume that the number of memorised patterns (p) is proportional to N:

$$p=\alpha N. \tag{3}$$

This problem has already been solved for the random diluted model (Derrida *et al* 1987). They found that for $\alpha<\alpha_1=(2/\pi)\,(1-Q)^2$ the system was able to retrieve and distinguish between the two patterns while for $\alpha_1<\alpha<\alpha_2=(2/\pi)\,(1+Q)^2$ it could retrieve but not distinguish between them. For $\alpha>\alpha_2$ the system was overloaded and could not retrieve any pattern.

As for Hopfield's model since there is only a finite number of correlated patterns the mean field equations are straightforwardly obtained from equations (3.5), (4.2) and (4.4) of Amit *et al* (1987):

$$m_1\pm m_2=(1\pm Q)\,\mathrm{erf}[(m_1\pm m_2)/(2\alpha r)^{1/2}] \tag{4a}$$

$$C=(2\pi\alpha r)^{-1/2}\{(1+Q)\exp[-(m_1+m_2)^2/2\alpha r]+(1-Q)\exp[-(m_1-m_2)^2/2\alpha r]\} \tag{4b}$$

$$r=(1-C)^{-2} \tag{4c}$$

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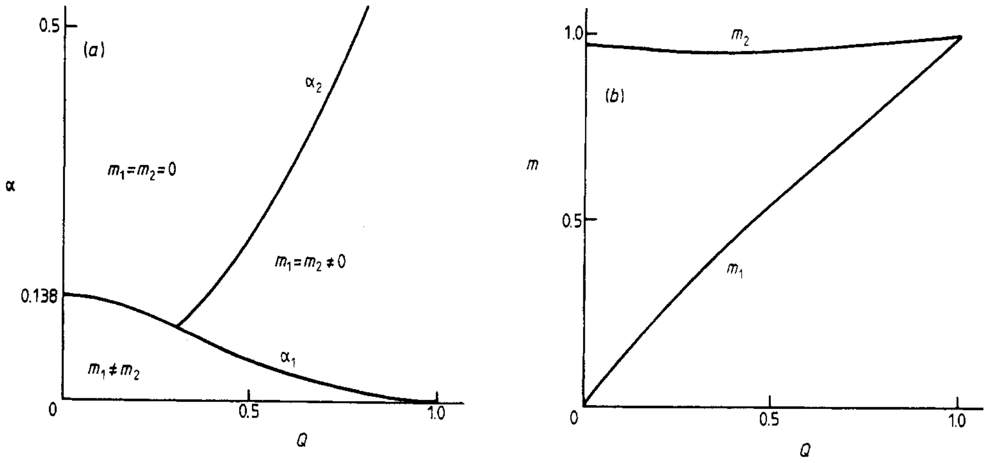

Figure 1. (a) Phase diagram presenting the three phases of the system. The curves α_1 and α_2 intersect at $Q=0.3048$, $\alpha=0.0954$. (b) The retrieval quality along curve α_1.

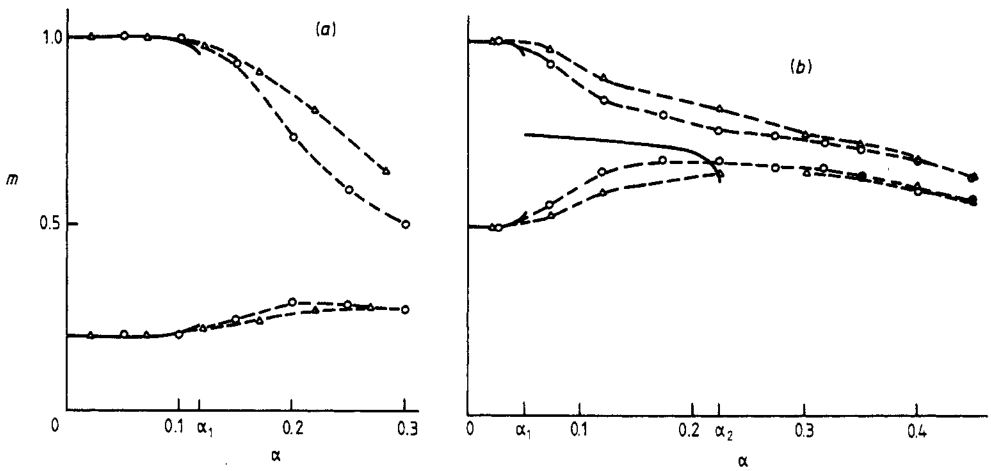

Figure 2. Retrieval quality as a function of α for (a) $Q=0.2$ and (b) $Q=0.5$. The simulations were carried out averaging each point over 3000 input patterns which were chosen to be one of the correlated ones, for $N=100$ (—△— —△—) and $N=200$ (—○— —○—).

where m_1 and m_2 are the order parameters

$$m_l=\frac{1}{N}\sum_{i=1}^{N}S_i\xi_i^l \qquad l=1,2. \tag{5}$$

The phase diagram consisting of three phases is shown in figure 1(a) where all transitions are first order. The curve α_2 which separates the phase $m_1=m_2\neq0$, where the system cannot distinguish the two patterns, from the spin-glass phase ($m_1=m_2=0$) is given by

$$\alpha_2=(2/\pi)[(A-1)/2+(A+1)Q/2]^2\approx(2/\pi)(0.0631+1.0631Q)^2 \qquad Q\gtrsim0.305 \tag{6}$$

with $A=(1+2y^2)\,e^{-y^2}$ obtained from the condition $2(1+y^2)\,e^{-y^2}[\pi^{1/2}\,\mathrm{erf}(y)]^{-1}=1$. The retrieval quality along α_2 is given by

$$m_1^c = m_2^c \approx 0.414(1+Q). \tag{7}$$

The increase of α_2 with Q was already expected since storing several times the same pattern or part of it is a very efficient mechanism to enhance its retrieval (Fontanari and Köberle 1988).

The curve α_1 bounds the phase $m_1 \neq m_2$ where the system is able to distinguish the two patterns. The retrieval quality along it is shown in figure 1(b).

In figure 2 we present the retrieval quality in the one- and two-transition regions together with the results of simulations for $N=100$ and 200. The values of α_1 agree rather well with the simulations while α_2 (figure 2(b)) seems to be undervalued.

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